waves, the lateral waves, or even the scattered radiation (far) fields due to incident surface or lateral waves.

This work is of particular interest in propagation problems when either the transmitter or receiver are near the irregular boundary. It is applicable to problems of coupling into and out of surface-wave structures. The electromagnetic problems considered here and the dual problems in acoustics are relevant to geophysical prospecting and active remote sensing.

ACKNOWLEDGMENT

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Traveling Waves in Coupled Yagi Structures

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Abstract—**The propagating mode in a coupled Yagi-Uda array of cylindrical wires is studied. The current distribution in each element, the phase velocity; and the cutoff frequency of the propagating mode are found, firstly by a numerical method and secondly by a method based on an assumed current distribution. These two methods yield essentially the same results. Mutual coupling between the arrays is studied. The characteristics of the propagating waves in the coupled Yagi-Uda structure have been measured. The experimental K-β diagram of the waves is obtained and is found to be in good agreement with the theory.**

I. INTRODUCTION

A TRAVELING WAVE can be supported on a periodic array of identical wires or strips that are equally spaced and perpendicular to the direction of the array. The existence of the traveling wave in such a structure, known as an infinitely long Yagi-Uda array, has been confirmed by theory and by experiments [1]-[5]. The traveling wave is a slow wave, that is, the phase velocity is smaller than the velocity of a uniform plane wave in the same medium in the absence of the structure. When one of the elements of the array is excited, currents on the parasitic elements are induced by a mutual coupling effect, resulting in a traveling wave. These currents have progressive phase shifts. The amount of phase shift and the distribution of the current in each element are determined by the array geometry. This information is useful when the structure is employed for uses such as millimeter waveguides or antennas [6].

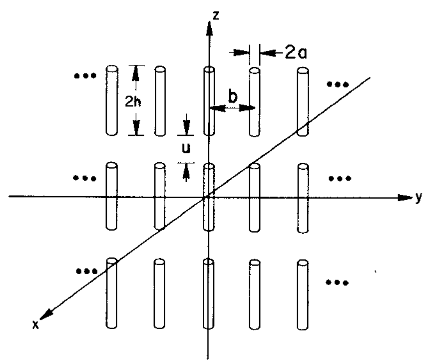

Fig. 1. Coupled three-row Yagi-Uda structure.

In order to carry more power or to divide it equally among several branches when the structure is used to transmit millimeter waves, several similar ones may be arranged in parallel in the same plane, as shown in Fig. 1. When used as an antenna, this arrangement could produce an elevation beamwidth that is narrower than that of a single Yagi array.

The present study also indicates the level of mutual coupling between two closely spaced Yagi structures when they are used separately for millimeter-wave transmission.

Little work has been done on the subject of coupled Yagi arrays. A study was made not long ago to obtain the phase velocity of a traveling wave on two Yagi arrays arranged in

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the same plane [2]. A disadvantage of this structure is that the transmission line that excites it is not placed at a neutral plane. So, the transmission line will radiate [7]. A three-row Yagi array structure does have a neutral plane. Thus the transmission line which excites it produces very little radiation.

II. Integral Equation and Solution

The coupled Yagi–Uda structure is shown in Fig. 1, where the physical dimensions of the array (a, b, u, and h) are defined. Defining ϕ as the incremental phase shift of the currents in adjacent elements, the current distributions on the wires at $y = 0$ are denoted by $I_2(z)$, $I_1(z)$, $I_3(z)$; at $y = \pm b$, $I_2(z)e^{\pm i\phi}$, $I_1(z)e^{\pm i\phi}$, $I_3(z)e^{\pm i\phi}$; at $y = \pm 2d$, $I_2(z)e^{\pm i2\phi}$, $I_1(z)e^{\pm i2\phi}$, $I_3(z)e^{\pm i2\phi}$; and so on. Assuming the array is made of perfect conductors, the tangential E field vanishes on the surface of each conductor. An integral equation is thus obtained:

$$\int_{-(U+3H)}^{-(U+H)} dZ'K(Z-Z')I_3(Z') + \int_{-H}^{H} dZ'K(Z-Z')I_1(Z') + \int_{(U+H)}^{(U+3H)} dz'K(Z-Z')I_2(Z')$$

$$= \begin{cases} l_3 \cos Z, & \text{for the inner element} \\ l_1 \cos Z + l_2 |\sin Z|, & \text{for the outer elements} \end{cases} \quad (1)$$

where $H = kh$, $z = kz$, $U = ku$, $k = 2\pi/\lambda$, and λ is the wavelength in the medium in which the structure is located. The constants l_1, l_2, and l_3 depend on the strength of the traveling wave. The kernel $K(z)$ is given by

$$K(z) = \sum_{n=-\infty}^{\infty} e^{i(n\phi + kr_n)}/r_n$$

$$r_n = [a^2 + (nb)^2 + z^2]^{1/2}. \quad (2)$$

The time variable $e^{i\omega t}$ is assumed.

The boundary conditions on the currents are

$$I_2[(U+3H)] = 0 = I_3[-(U+3H)]$$
$$I_2[(U+H)] = 0 = I_3[-(U+H)]$$
$$I_1[H] = 0 = I_1[-H]. \quad (3)$$

The integral equation (1) is solved by two methods. In the first, the unknown currents I_1, I_2, and I_3 are treated as column matrices and the integral equation is solved by standard moment methods [8]. In the second, the current distribution is assumed to be a superposition of a set of specially chosen functions. On the inner element, three-term theory [9] is applied. The current distribution is approximated by the sum of a shifted cosine distribution and a half-cosine distribution. On the outer elements, an antisymmetrical component is added. In summary, the currents are represented by

$$I_1(Z) = A_0[\cos Z - \cos H] + B_0\left[\cos\frac{Z}{2} - \cos\frac{H}{2}\right]$$

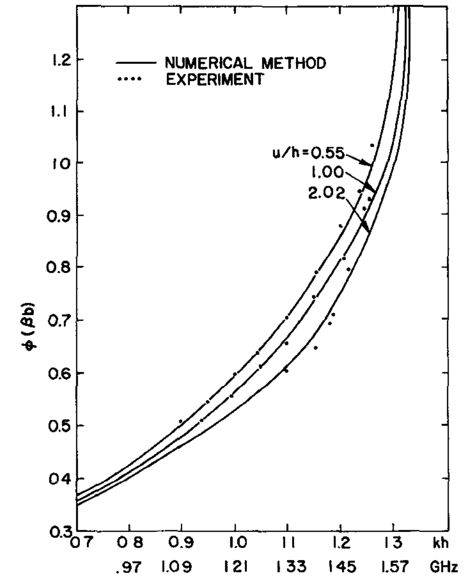

Fig. 2. K–β diagram for $a/h = 0.03$ and $b/h = 0.49$ with different separations ($a = 0.125$ cm, $b = 1.95$ cm, and $h = 3.95$ cm).

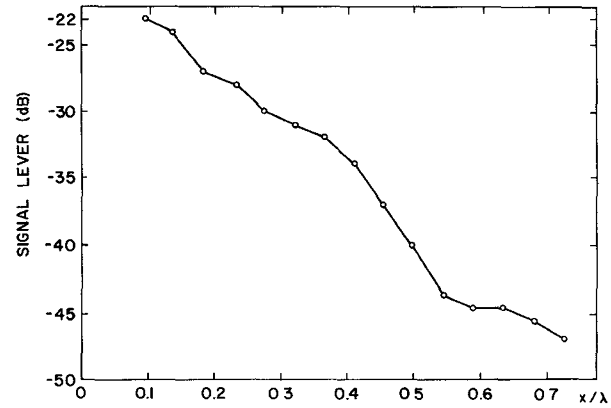

Fig. 3. Measured field intensity in transverse direction x/λ away from the wire of the middle array for $u/h = 0.55$ at $f = 1.35$ GHz ($\lambda = 22.2$ cm).

for the inner element

$$I_2(Z) = C_0[\cos(Z - 2h - U) - \cos H] + D_0\left[\sin(Z - 2H - U) - \frac{Z - 2H - U}{H}\sin H\right] + E_0\left[\cos\left(\frac{Z - 2H - U}{2}\right) - \cos\frac{H}{2}\right]$$

where $I_3(Z) = I_2(-Z)$, for the outer elements; and A_0, B_0, C_0, D_0, E_0 are the unknown constants. By requiring the boundary condition to be matched at eight points on the wires, (1) is converted into a matrix equation and is then solved.

III. Numerical Results and Discussion

The two methods described in the previous section yield essentially the same numerical results. Detailed discussion and comparison of the results are given elsewhere [10]. A typical K–β diagram is shown in Fig. 2. It is seen that as the arrays are placed closer to each other, the passband of the traveling wave is narrower, and the phase velocity, which is related to Φ by $V_p = (kh/\Phi)(b/h)V_0$, is lower. Comparison with the result obtained for a single-row Yagi structure shows that the coupling effect at $U = 2h$ causes less than a 2-percent change in the K–β diagram.

The theoretical result has been verified by experiments [10], as shown in Fig. 2. Theory also predicts that the field intensity in the transverse direction (x direction in Fig. 1) decays exponentially. This is a typical characteristic of a guided wave. In Fig. 3 the measured field intensity is shown to decay at approximately 35 dB per wavelength in the x direction. This shows that the level of mutual coupling in the transverse direction is rather low when the arrays are separated by at least one wavelength.

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Coupling of Circuit Structures to Magnetostatic Modes of Ferromagnetic Resonators

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***Abstract*—The coupling between a current-carrying circuit structure and the magnetostatic modes of a general ferromagnet, such as a YIG resonator, is examined. A small-signal theory is presented that describes the excitation of an arbitrary mode in terms of an effective susceptibility matrix; this description leads to a simple method for calculating the z parameters of the resonator and coupling structure combination. This result is tested by comparison with other theory and with experiment. Applied to the case of a uniform field exciting the uniform mode of an ellipsoidal resonator, it reduces to Carter's well-known formula. Applied to the case of a particular nonuniform field exciting the main mode of a thin square resonator, it predicts the experimental finding that the coupling strength depends only on the resonator's thickness. This last case illustrates the extended generality of our result which allows the treatment of situations where the RF magnetization and field are not uniform.**

Nomenclature

$\hat{a}_m$	Unit vector in the direction of dc magnetization.
$\hat{a}_x, \hat{a}_y, \hat{a}_z$	Coordinate axis unit vectors.
H	Total magnetic field.
h_d	RF demagnetizing field.
h_e	Circuit induced field.
H_i	Magnitude of dc field inside resonator.
H_0	Magnitude of applied dc magnetic field.
k_p	Magnetic coupling function for port p.
k_{pu}	Decomposition of k_p on Ψ_{ur} and Ψ_{ui}.
$k_{pu}^{\dagger}$	Adjoint of the matrix k_{pu}.
m	RF magnetization.
M	Total magnetization.
M_s	Saturation magnetization.
m_u	Complex amplitude of the uth eigenfunction.
S_Ω	Surface enclosing the volume Ω.
V	Resonator volume.
γ	Gyromagnetic ratio.
δ_{uv}	Kronecker symbol.
χ_u	Matrix of susceptibilities for the uth mode.
Ψ_u	uth eigenfunction.
Ψ_{ur}, Ψ_{ui}	Normalized real and imaginary parts of Ψ_u.
$\Delta\omega$	Unloaded radian bandwidth of resonator.
ω_M	Magnetization frequency γM_s.
Ω	All space excluding metallic conductors.

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Introduction

A FERROMAGNETIC resonator was first used in a practical circuit—a YIG sphere filter gyrator—by Degrasse [1]. The problem of coupling a circuit, via induced magnetic fields, to an ellipsoidal resonator was analyzed